Conditions and Constraints for better Approximate Reasoning with Fuzzy Logic

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Abstract. Fuzzy sets and fuzzy logic were proposed as a means to handle states and/or relations that are not well defined or that present gradual transitions. These characteristics have enabled simpler solutions to numerous problems, particularly in the area of control. Nevertheless, the model used for most applications does not fully match the requirements for fuzzy logic. Though the model is fuzzy, because it is based on fuzzy sets and fuzzy operators, it is not a logic model. Also, fuzzy models are generally more important from a numeric approximation point of view. In this paper we give some conditions and constraints for better approximate reasoning with a fuzzy logic model, which is more important from a linguistic approximation point of view. However, the monotonic property is necessary, otherwise, when this condition is not fulfilled, it can be used as exceptions or as approximate reasoning with decision making.

Keywords: Fuzzy Logic, Approximate Reasoning, Fuzzy Rules, Human-Machine Interface.

1 Introduction

Since fuzzy sets and fuzzy logic were proposed, they have been used as an excellent tool to handle states and/or relations that are not well defined or have gradual transitions among other characteristics [1, 2]. These characteristics enabled simpler solutions to numerous problems, particularly in the area of control [3, 4]. Nevertheless, for most applications the model does not fully match the requirements of fuzzy logic. Therefore, the model is fuzzy, because it is based on fuzzy sets and fuzzy operators, but it is not logic because the inference with these models differs from the inference in logic. Besides this, these models are generally more important from a numeric approximation point of view [5].

A strictly fuzzy logic model has little application even if some efforts have been made to use it as it has been used for fuzzy models in control. The main problem arises at the moment of approaching either the conclusion of the fuzzy relation representing the fuzzy rules, or approaching the conclusions which result from the application of the fuzzy rules individually. In both of these cases, it generally results in a subnormal conclusion with a significant degree of uncertainty for a complete rule set, resulting in a conclusion that generally has little meaning. Obviously, because of this, it is difficult to establish a complete set of fuzzy rules that satisfy all the conditions

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required for inference, and that allow obtaining the maximum significance for the conclusions of the fuzzy logic model.

This problem of the fuzzy logic model can be explained in the following way. When two fuzzy rules are fired at the same time, but these rules contain contradicting conclusions, the result could be an empty conclusion. However, the necessary conditions required for the parameters of the fuzzy logic model, as well as the restrictions on the maximum uncertainty on the input values to the model, can be very well established. This model can be used for approximate reasoning, and, as more knowledge is involved (more rules are fired at the same time), the conclusion is more precise [6, 7].

In this paper a fuzzy logic model is considered. We show that with this model the most important aspect is the inference using linguistic terms, which serve as labels of the fuzzy sets. Under these conditions, and if the model is well designed, the gradual transition in the conditions of the rules helps to obtain a gradual transition on the terms involved in the conclusions. This avoids the uncertainty other than the fuzziness. This is a natural situation because the uncertainty in the fuzzy rules and in the input data have been taken into account in advance, so better conclusions result from complete input data and from the maximum amount of knowledge (rules) involved in their calculations.

As is shown in this work, the fuzzy sets of the input variables must be defined according to the input data. If they consist on precise values only, the fuzzy sets can be defined as a strict possibilistic partition. Otherwise, for fuzzy input data, the partition must take into account the maximum fuzziness or imprecision in order to define the fuzzy partition of the inputs. We also consider the case of incomplete input and we propose a non standard procedure for the calculation of the output.

One necessary condition, in order to obtain the previous results, is that the model must be structured for monotonic knowledge. When this condition can not be satisfied, because there are rules with similar conditions but very different conclusions, we propose the use of rules that do not satisfy the monotonic property as exceptions. For this, we handle them in a different way, keeping monotonic knowledge in one fuzzy logic model and exceptions in another model, such that the conclusion of the rules are: *y is B unless y is* β *C*, where *y is* β *C* represents an exception and β it the degree of certainty. So, the goal of this work is to present the conditions and constraints in order to use the fuzzy logic model for better approximate reasoning.

Section two presents the fuzzy logic model, and in section three we use this model to do approximate reasoning. As this model has monotonic rules, we consider the linear and nonlinear cases as well as the precise and fuzzy inputs, and the case of incomplete input. In this section we also handle as exceptions the rules that do not satisfy the monotonic conditions and we propose a more general output for a fuzzy logic model. Finally, in section four, we give the main conclusions concerning this work.

2 Fuzzy Logic Model

Fuzzy logic was proposed as a means to do reasoning under uncertainty. This can be done with a fuzzy model based on a set of conditional fuzzy rules such as *"if ... then ..."*. The quantification of the condition is made with the observation *x is A'* and the inference follows the Generalized Modus Ponens (GMP) [2], such that a conclusion *y is B_j'* results for each rule, as shown in the next scheme.

Rule:	If x is A_i	Then y is B_j
Observation:	x is A'	
Conclusion:		y is B'

A rule is modeled by a relation that depends on an implication operator. In the case of fuzzy logic there exist several possibilities and the choice can be made by the properties and/or the behavior of the operator [8]. In fact, this has resulted in a typology of fuzzy rules [9]. In this work we consider certainty rules, which express that y is in B_i only for input values x in the core of A_i.

From the computational point of view, a fuzzy rule is modeled by a fuzzy relation $R_k = A_i \rightarrow B_j$, where A_i and B_j are the fuzzy sets of the input and output variables respectively. The calculation of a conclusion B' using this relation is $B' = A' \circ R_k$, that is, it is calculated according to $\mu_{B'}(y)=sup_{x\in X}T[\mu_{A'}(x),\mu_R(x,y)]$, $\forall y \in Y$, where the relation $\mu_R(x,y)$ represents the conditional rule, and T a T-norm. Fig. 1 shows an example of fuzzy rule based on the Kleene Dienes implication ($\mu_R(x,y)=min(1-\mu_A(x), \mu_B(y))$).

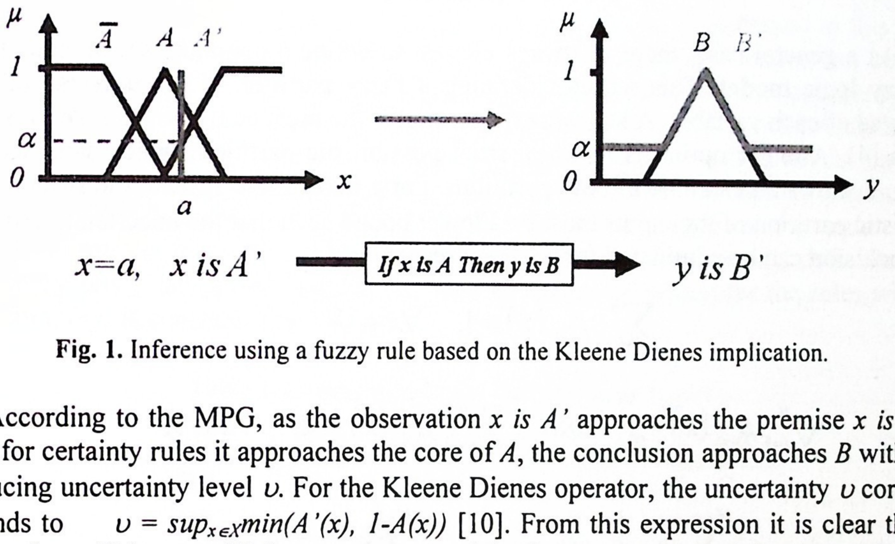


Fig. 1. Inference using a fuzzy rule based on the Kleene Dienes implication.

According to the MPG, as the observation *x is A'* approaches the premise *x is A*, and for certainty rules it approaches the core of *A*, the conclusion approaches *B* with a reducing uncertainty level υ. For the Kleene Dienes operator, the uncertainty υ corresponds to $\upsilon = sup_{x\in X}min(A'(x), 1-A(x))$ [10]. From this expression it is clear that this value will be zero iff the complement of the fuzzy set $A(x)$ is defined outside of the support of $A'(x)$, or $A'(x) \subseteq core\ of\ A(x)$.

From the logical point of view, given A_k the property $B_k = \cap_{i=1,n} A_k o(A_i \rightarrow B_j)$ must hold for a set of implicative fuzzy rules. And, if A_k is defined such as $A_k \subseteq A_i \cap A_{i-1}$, the conclusion must be a logical output satisfying the previous property where $B_k \subseteq B_j \cap B_{j-1}$, according to the Generalized Modus Ponens.

In other words, take A_k such that the membership degree to A_i is equal to one, and the membership degree to A_{i-1} is equal to α, with $\alpha \in (0,1)$, then the conclusion is in this case $B_k = \cap_{i=1,n} A_k o(A_i \rightarrow B_j)$, with $B_k \subset B_j$. The degree of approximation between these fuzzy sets depends on the α value. For example, as $\alpha \rightarrow 0$, $B_k \rightarrow B_j$, whereas, as $\alpha \rightarrow 1$, $B_k \rightarrow B_j \cap B_{j-1}$.

As can be seen in Fig. 1, the conclusion B' is a normal fuzzy set similar to B, and it presents the uncertainty υ over all the universe of discourse. The shape of B' and its similarity to B change according to the similarity between A and A' and the implication operator used. Also, as the final conclusion must be determined by the conjunction of the individual conclusions, this conclusion will be normalized only if the fuzzy partition of the output was built in order to obtain this result [7].

When using a fuzzy logic model we can follow two approaches. One of the approaches is to combine in a unique fuzzy relation the relations given by the individual fuzzy rules as given in (1) and (2). The other approach is to combine the results given by inference with individual fuzzy rules, as given by (3). The main differences between both approaches are the operating mode and the final conclusion. For the former model, the conclusion is more precise [11]. However, in this work we limit the analysis to models with individual rules.

$$R = \cap_{k=1,n} R_k \tag{1}$$

$$B' = A' \text{o} R \tag{2}$$

$$B' = \cap_{k=1,n} A' \text{o} R_k \tag{3}$$

As a general case, there is always interest to define a complete set of rules for a fuzzy logic model. This requires defining a fuzzy partition of the universe of discourse of each variable. A strict fuzzy partition is the most common case, and it satisfies (4). Another option is to use a strict possibilistic partition, which satisfies (5). Fig. 2 shows an example of both partitions. For a fuzzy logic system, the strict possibilistic partition of the inputs must be a lower bound such that the uncertainty υ of the conclusion can be eliminated for coherent reasoning.

$$\sum_{i=1}^{n} \mu_{A_i}(x) = 1, \quad \forall x \in X \tag{4}$$

$$\vee_{i=1}^{n} \mu_{A_i}(x) = 1 \quad and \quad \sum_{i=1}^{n} \mu_{A_i}(x) < 2, \quad \forall x \in X \tag{5}$$

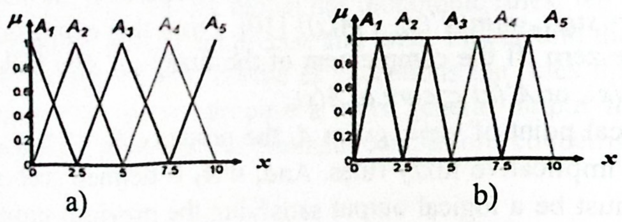


Fig. 2. a) Strict fuzzy partition, b) Strict possibilistic partition.

According to the previous analysis of the model, the fuzzy partition of the inputs must be made such that they allow better conclusions and a minimum of uncertainty in the result. Comparatively, the intersection of two adjacent fuzzy sets must be placed at the invariants, or at the local minima, in a similar way as a fuzzy model built for function approximation [12]. On the other hand, the fuzzy sets of the output must have at least the minimum intersection such that every conclusion given by the model is normalized. In other words, if p is the maximum number of different conclusions given by rules that could be activated at the same time, this is the minimum number of output fuzzy sets that must have a non empty intersection in order to satisfy this condition.

3 Approximate Reasoning

As it was shown in [7], the fuzzy model must have possibilistic partitions in the input universe of discourses, but the output needs a partition that satisfies the minimal number of intersection fuzzy sets. In this work we consider more restrictions and conditions in order to take better advantage of the fuzzy logic model. These results improve some results presented in [13, 14], and generalize to the multi-input model. For instance, take the next rule as an example of fuzzy rule

$$\textit{If } x_1 \textit{ is } A_i \textit{ and } x_2 \textit{ is } B_j \textit{ Then } y \textit{ is } C_k \quad (6)$$

where $i = 1, 2, \ldots, N_1, j = 1, 2, \ldots, N_2$, and $k = i+j-1$. The rules expressed in this way refer to monotonic knowledge so the relation among inputs and output can be expressed as a table of rules, as shown in Table 1. In order to simplify the analysis of the model we consider a monotonic reasoning. When this condition is not satisfied, the non-monotonic part is taken into account in a parallel model with a model having monotonic knowledge and another model with the rules that do not satisfy monotonic condition. For example, take the previous rule which supposes the order $C_{k-1} < C_k < C_{k+1}$ on the fuzzy sets of the output. Under this consideration, the definition of the fuzzy sets is simplified because we have the same conclusion for the rules where the sum $(i+j)$ is constant. See Table 1.

Table 1. Monotonic rules for the fuzzy logic model.

$X_2 \backslash X_1$	BN	SN	ZE	SP	BP
BP	C_5	C_6	C_7	C_8	C_9
SP	C_4	C_5	C_6	C_7	C_8
ZE	C_3	C_4	C_5	C_6	C_7
SN	C_2	C_3	C_4	C_5	C_6
BN	C_1	C_2	C_3	C_4	C_5

The monotonic condition, accompanied with the design of the fuzzy sets for the model, is a necessary and sufficient condition in order to have coherent reasoning. If this condition can not be satisfied, it is necessary to divide the model into sub-models and to build a meta-model such that it will be the responsible for determining the final output, according to the output provided by each model, or if it is possible to consider the non monotonic behavior as exceptions as proposed below.

Taking into account the previous considerations, the necessary fuzzy sets for the output can be calculated as $n_z=(1-N)+\Sigma_{j\ 1,N} n_j$, for N inputs, where n_z and $n_j, j=1,2$, ..., N, are the number of fuzzy sets for the output and the inputs respectively. So, let us consider a model with two inputs having strict possibilistic partitions. Every input value has no more than two non-zero membership degrees in accordance with fuzzy sets. Take i and $i+1$ for input one, and j and $j+1$ for input two for example. Then, the maximum number of rules activated at the same time is four, but their conclusions use only three fuzzy sets, that is $k_1=i+j-1$, $k_2=i+j$ and $k_3=i+j+1$.

An example of fuzzy sets for this model is given in Fig. 3. When a strict possibilistic partition is used in the inputs, the model is designed in order to work only with precise values. In this case, in the partition of the output there must be the intersection of at least three adjacent fuzzy sets. This implies that the maximum number of rules fired at the same time is four but the number of different conclusions is three.

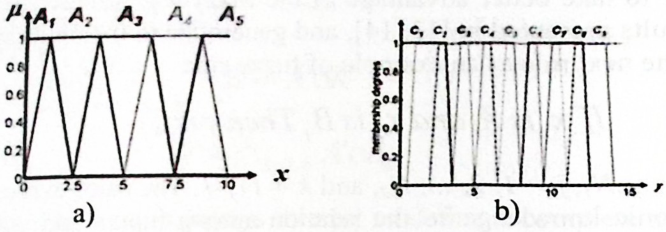


Fig. 3. a) Strict possibilistic partition for the inputs. b) Possibilistic partition for the output.

Linear Reasoning

According to the condition for the fuzzy sets of the output, and as the model is more appropriate for an approximate reasoning based on symbols, the partition of the output must be defined based on the intersection of the fuzzy sets. This is shown in Fig. 4a, where two hypothetical fuzzy sets (C_0 and C_{10}) are considered at the ends in order to define completely the fuzzy sets. They provide the internal limit of the C_2 and C_8 fuzzy sets.

Based on these fuzzy partitions, the fuzzy logic model gives approximate reasoning for three kinds of symbols as those illustrated in Fig. 4b. This also shows that the most precise linguistic conclusions of the model are the fuzzy sets $C_i \cap C_{i+1} \cap C_{i+2}$. Take for example the conclusions of two individual rules such as C_3 and C_4. Their aggregation is a normal fuzzy set. If the conclusion is given by four fuzzy rules, say C_1, C_2, C_2 and C_3 for example, the result continues to be a normal fuzzy set. If only one rule is fired and the conclusion is C_7 for instance, the result is normal and more imprecise. The main difference between these examples is that when more knowledge is taken into account, more rules are fired, and the more precise the final conclusion becomes.

Linear reasoning is considered a fuzzy logic model with uniform strict possibilistic partition of the inputs, and uniform possibilistic partition based on the most precise fuzzy sets of the output, as those of the Fig. 4a. Fig. 5 shows an example. So, as more rules are involved in the final conclusion, they serve as a factor to increase, to some degree, the precision of the final conclusion. The increase in precision depends on the certainty of the activated rules.

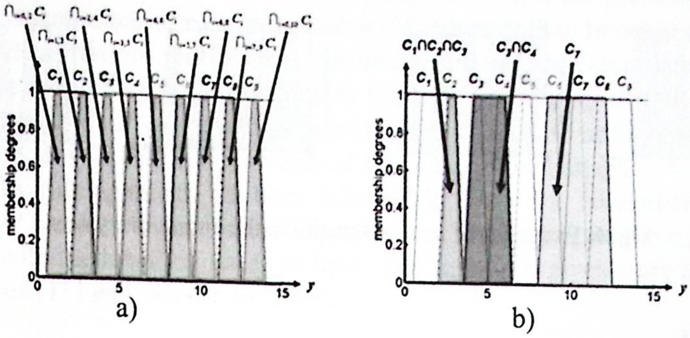


Fig. 4. a) Most precise fuzzy sets that defines the partition of the output. B) Different fuzzy sets that can be given as result of reasoning.

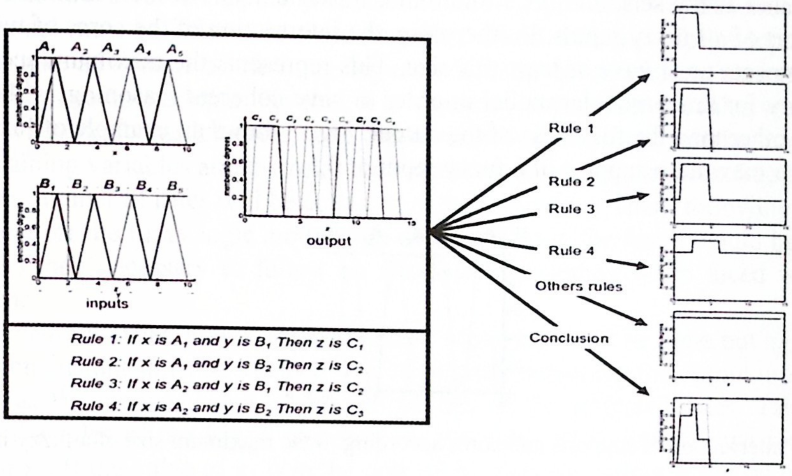
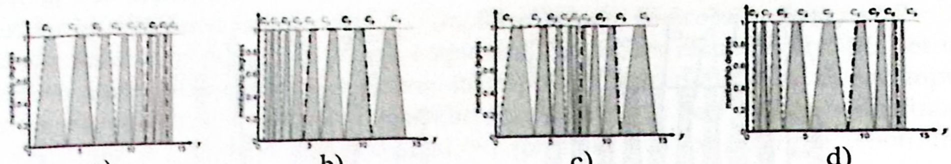


Fig. 5. Fuzzy logic model with linear reasoning.

Nonlinear Reasoning

In the previous section we consider monotonic and linear reasoning. However, fuzzy logic is interesting because nonlinearity can be handled in a natural way. This can be observed in the perception, reasoning and decision making of human beings, and this is what fuzzy logic was developed to model [15]. So, if fuzzy rules are defined according to the monotonic property, the most difficult task is to find the appropriate distribution of the fuzzy sets. The fuzzy sets of the inputs and the output must take into account the nonlinearity of the reasoning process in the space where the model is defined. Fig. 6 contains some examples for the output fuzzy sets $C_i \cap C_{i+1} \cap C_{i+2}$. In Fig. 6 there are some regions with more precise and some regions with more imprecise information, the former represented by narrower fuzzy sets. Obviously, the nonlinearity of reasoning can also be done by the non uniformity of the partition of the inputs. Nevertheless, they are not shown here for simplicity.



a) b) c) d)

Fig. 6. Examples of non uniformity in the output fuzzy sets.

Fuzzy Inputs

The partition on the inputs depends on the application. For precise inputs the best is a strict possibilistic partition, whereas for fuzzy inputs the partition must not be strict. Although the core of each fuzzy set must have a minimum intersection with the core of other fuzzy sets, and, as a minimum, must be equal to the maximum size of the support of all fuzzy inputs. Furthermore, the intersection of the cores of two adjacent fuzzy sets must have at least this size. This represents the maximum acceptable uncertainty for this particular model in order to have coherent reasoning, without uncertainty other than the fuzziness of the output. Fig. 7 shows an example of fuzzy partition for a maximum support of a fuzzy input of 0.5.

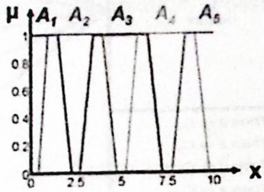


Fig. 7. Intersection of supports and cores according to the maximum size of a fuzzy input.

Consider the maximum imprecision allowed at the inputs. As shown in the Fig. 7, the minimum separation between the supports of two non-adjacent fuzzy sets of the inputs must then be this maximum imprecision. Thus, for every input value at most two fuzzy sets have a non-empty intersection. Also, the intersection of the cores of two adjacent fuzzy sets must be equal or greater than the maximum imprecision.

Taking into account the uncertainty (fuzziness or imprecision) of the input, the fuzzy sets of the inputs can be defined according to this uncertainty. However if the input is within the set of elements of only one fuzzy set, or is within the core ($A_i \cap A_j$), there will be no distinction among precise, imprecise and fuzzy values of the input. This can be interpreted as a degree of distinction which has a strict possibilistic partition for one side (total distinction), and a qualitative partition for the other side (zero distinction). Besides, as the input takes on a fuzzier or a more imprecise value, the model generally gives a more imprecise output.

Designing the partition of the inputs for a maximum fuzziness of the data, each of the inputs will activate a maximum of two rules. Otherwise, if we allowed the input to activate three fuzzy rules or more per input, the conclusions will be much more imprecise and it will require the intersection of four or more fuzzy sets. Thus, this will require bigger supports and cores for these fuzzy sets. So, if the previous condition is not satisfied, the number of rules activated will be greater than two and it is necessary to make some restrictions on the fuzzy partition of the output. Nevertheless, it is more difficult to satisfy this requirement than to limit the uncertainty of the inputs. Under these conditions $A_i \cup A_j$ can not be accepted as input, because in this case the conclusion of the model will be Y (the universe of discourse of the output).

The rule-by-rule approach has been followed in this work. Nevertheless, if the inputs can have fuzzier values than anticipated, then first aggregate the rules and afterwards infer with the resulting fuzzy relation. This approach gives more precise results for these cases [11] and this can be helpful.

Incomplete Input

As previously shown, for complete input, the conclusions become more precise, as more rules are involved in the particular reasoning process. Nevertheless, a fuzzy logic model must be able to do reasoning when faced with incomplete input. A proposal to take advantage of the fuzzy logic model is to do incomplete reasoning when faced with incomplete input. For this, it is necessary to consider the rules while ignoring the variable for which there is no input value available, and to take into account the remaining variables and the rules that contain them in the premises. So, given this, a greater number of rules will be activated at the same time. Then, following the same procedure for the fuzzy logic model with complete input, the result would be incoherent. It is then necessary to follow an alternative procedure when faced with these situations.

For example, when inputs are incomplete, reasoning must be done but aggregation of the conclusions must be avoided. For incomplete reasoning the output must be calculated as the median value of the conclusions of the activated rules. Take for instance, the rules of the Fig. 8 and suppose the value of x_2 is unknown for a given value of x_1. If the value of x_1 is in the core of A_1, then the output is the set of conclusions C_1 to C_5. The median value, in this case, is C_3. If the value of x_1 is in the core of A_2, then the conclusions is the set C_2 to C_6. The corresponding median value is C_4. However, if the input value is in the core of A_1 and A_2, then the set C_1 to C_6 is given as the conclusion. The median value in this case is C_3 *and* C_4 which can be translated into a mathematical expression as $C_3 \cap C_4$. This can be the basis for an iterative process of looking for a good conclusion through an approximate reasoning process, waiting for the missing input values. This could also be helpful for the application of the fuzzy logic models in research engines.

As can be seen, the monotonic behavior of the model is not lost when faced with incomplete input. What are lost are the most precise symbols of the output, because in the previous example the conclusion could be two kinds of fuzzy sets but not the most precise ones, as shown in Fig. 4b. So, for these conditions the output can be calculated as the median value that corresponds to a fuzzy set of the output for an odd number of conclusions, and an intersection of two fuzzy sets for an even number of conclusions.

However, the model can still be useful in situations where data is incomplete. For example, for a two input and one output model, if we have an input value for one variable but the value for the other variable is not available at the moment of the inference, it seems interesting to obtain an output (to take advantage of the knowledge coded in the fuzzy model), and to obtain an output that is a suggestion, not a conclusion. Consider a model with three inputs,

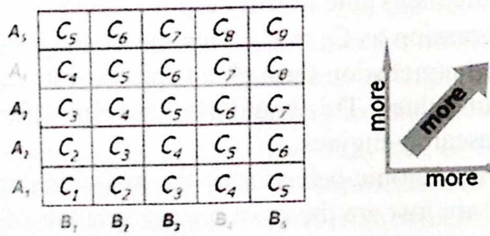
$$If\ x_1\ is\ A_i\ and\ x_2\ is\ B_j\ \ and\ x_3\ is\ C_k\ Then\ y\ is\ D_m \tag{7}$$

where $m = i+j+k-2$. If only x_1 is available, the conclusion is $y = \{D_i, D_{i+1}, \ldots D_{i+N2+N3-2}\}$. The final result can be approached following the previous procedure. When two inputs are available, the conclusion is $y = \{D_{i+j-1}, D_{i+j}, \ldots D_{i+j+N3-2}\}$. Finally, consider the values when all three inputs are available. In this case, the conclusion is $y = D_{i+j+k-2}$. This example clearly shows that when the input information is more complete, the conclusion is more precise.

Non monotonic Reasoning

Monotonic reasoning, linear or nonlinear, presents a limitation as it does not offer much flexibility in the definition of the fuzzy rules, as shown in Fig. 8. In this situation the conclusions of adjacent rules must have a normal intersection, so the implemented reasoning is of the type "More (Less) … More (Less) …" or any combination of these. Under this point of view, the fuzzy logic model seems to be a detailed description of a gradual rule "More … More …". In order to relax the condition of adjacent conclusions for adjacent rules, the fuzzy sets of the output are harder to define and it becomes more complex as further fuzzy sets must have a non-empty intersection.

Considering regions that go beyond the region where a fuzzy logic model is defined, we can find each region with monotonic knowledge but with non monotonic knowledge among regions. A multi-model approach can be applied such that the behavior is well represented by fuzzy logic models. However, if a domain presents a monotonic behavior, except for particular cases that do not satisfy this condition, the model can be extended to a model with exceptions and the conclusions of the rules modified such that "*y is B unless y is βC*", where "*y is βC*" represents the exception to the monotonic condition, and β the certainty degree of the exception.



a) b)

Fig. 8. a) Rules for monotone reasoning b) Gradual reasoning more (less) … more (less) ..

The fuzzy rules of a model, including an exception, are represented in Fig. 9a, although this model can be decomposed into two fuzzy logic models in parallel. One of the models with a complete set of monotonic rules, as those of the Fig. 8, and the other with the exception as in Fig. 9b. The final output of the model will be the conclusion of the rules or the exception, depending on the value of the degree of certainty.

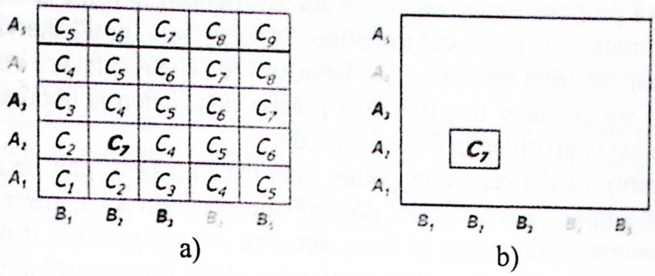


Fig. 9. Non monotony in a rule set.

Calculus of Linguistic and Numeric Output

The output of a fuzzy logic model approximates fuzzy sets with different precision; Fig. 4b shows some examples. However, depending on the application of the fuzzy logic model, it is necessary to approximate the output by the linguistic terms of the possible output; C_i, $C_i \cap C_{i+1}$, $C_{i-1} \cap C_i$, $C_{i-1} \cap C_i \cap C_{i+1}$, etc., and a set of qualifying terms, such as almost, approximately, more or less, etc., that grades the precision of the linguistic term with the conclusion of the model, such that the conclusion would be *almost* C_i, *almost* $C_{i-1} \cap C_i$, *approximately* $C_{i-1} \cap C_i$, *approximately* $C_{i-1} \cap C_i \cap C_{i+1}$, *more or less* $C_i \cap C_{i+1}$, etc. It could also be necessary to approximate the output by numeric values of the universe of discourse Y. A fuzzy logic model can provide both types of values although the linguistic output is the most interesting from the logic point of view.

The linguistic approximation can be used with other models or as a human-machine interface, whereas the numerical approximation can be used with models developed to control systems for example. The fuzzy logic model is then very useful for logical reasoning and numerical approximation, so the model offers the possibility of a good relationship between meaning and precision.

The numerical approach consists in the calculation of a precise value from the fuzzy conclusion of the model using a center of area defuzzification method for example [16]. Consider the fuzzy conclusion shown in Fig. 5. In this case, a defuzzification method, as the one mentioned previously, can be applied to calculate a numerical output for the model. From the design of this model it is possible to get a good numerical approximation, which shows that even if the fuzzy logic model is more appropriate for linguistic reasoning, it is able to easily interface with models where precision is required.

4 Conclusions

The main problem with fuzzy logic models is in the ability to obtain coherent outputs. In this paper we propose some conditions and restrictions in order to solve this problem. Indeed, a method is proposed to define the fuzzy sets for all the variables of the model, such that coherent reasoning can be achieved. The results are very interesting, and, overall, if we consider that the most precise conclusion is obtained when more knowledge (rules) is involved in the calculation.

The uncertainty of the reasoning, other than the fuzziness, can be avoided if the partitions of the inputs have a strict possibilistic partition as a lower bound. Also, normal conclusions, in the sense of fuzzy sets, are always possible if the partition of the output respects the minimum number of cores of the fuzzy sets that must intersect. Linear and nonlinear reasoning can be made with the fuzzy logic model according to the distribution of the fuzzy sets in their respective universe of discourse; however, obviously, linear reasoning gives the simplest partition and distribution.

Knowing the maximum uncertainty of fuzzy inputs it is possible to determine the parameters for the model, in particular, the fuzzy partition of the inputs, in order to obtain coherent reasoning for these kinds of inputs. Also, the knowledge coded in the model can be used when faced with incomplete input. Nevertheless, the model requires of a different procedure for the calculation of the output, and the result is considered a suggestion and not a conclusion.

Non-monotonic behavior is difficult to model with a fuzzy logic model, particularly for the aggregation of the conclusions of the rules. In this work it was proposed that the monotonic behavior should be kept in one model and the cases of non-monotonic behavior be kept in another model such that the final conclusion can be modified to *y is B unless y is* β *C*, which takes into account the monotonic behavior subject to the exception.

The fuzzy logic model is well adapted in order to make a good connection between models in different levels of a hierarchy, because approximate reasoning can be made from the linguistic or numeric point of view. Linguistic reasoning requires the approximation of the conclusion with the linguistic terms of the fuzzy sets, and a set of qualifying terms such as almost, approximately, more or less, etc. Linguistic reasoning can be used for a higher level of reasoning in an intelligent system, where meaning is more important than precision; this could be the case of a human-machine interface for example. The numeric reasoning requires a defuzzification method, as the center of area method, which could be applied to the conclusions so that precise values can be given and used as input to another fuzzy model. So, the fuzzy logic model described here provides a good relationship between meaning and precision.

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